

A SIMPLIFIED METHOD PROPOSAL FOR PRACTICAL DETERMINATION OF APERIODIC TWO-PHASE FLOW INSTABILITY

ENRICO LORENZINI

Faculty of Engineering, University of Bologna, Bologna, Italy

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Abstract—A short outline of the instability phenomenon is given. Two methods are explained for the problem solution, i.e. the practical determination of the existence or not of an aperiodic instability and, in the case of existence, what to do to eliminate it. The first method substantially comes from that used by boiler designers. A second, new method is suggested: this is a manual, simple method. The two methods are compared: the new one is reliable, and has the great advantage of the simplicity and the possibility of specifying the diaphragm at the pipe mouth for eliminating the two-phase flow instability.

INTRODUCTION

In two-phase flows, for particular conditions, periodic or aperiodic instabilities may occur (Tong 1965). Ledinegg established the criterion

$$\partial(\Delta P)\partial G < 0, \quad [1]$$

to establish the necessary analytic condition for the existence of aperiodic instabilities. For periodic oscillations, the criterion is not valid (Quandt 1961, Pulling & Collier 1963). Multipipe boilers with parallel waterpipes may have varying delivery capacities in their pipes, even though the pressure loss across the inlet and outlet manifolds is constant.

These effects are clearly undesirable because they affect the overall performance of the boiler. Tube failure may occur if the flow in any of the tubes is so low as to lead to dryout and consequent tube damage, or to turbine damage if the flow is so high as to give excessive carryover.

A similar phenomenon can happen in the steam generators of nuclear plants, although in this case it is extremely unlikely that there could be heavy tube damages as the heating fluid temperature is not high enough to damage the pipes of the exchanger. In this case also, the performance changes of the generator give rise to undesired results on plant regulation and thermal efficiency.

This instability phenomenon can also occur in reactor cores. Here, flow rate reductions in the channels can result in fuel element damage (Lorenzini 1976).

To understand the aperiodic instability phenomenon several theoretical and experimental researches have been carried out (Anderson & Lottes 1962, Bergles *et al.* 1967, Bouré *et al.* 1971, Ginoux 1978, Ishii & Zuber 1970, Tong 1968, Veziroglu & Lee 1971), particularly on single pipes which carry a two-phase mixture. In practice, however, one has generally to consider systems with several parallel pipes, where the thermo-hydraulic phenomenon is evidently more complex.

In this paper a simplified physical treatment of the phenomenon is given that aims to predict the occurrence of the instability; further, a simple method is suggested for the hand calculation of the regions of aperiodic instability.

Comparisons are made between this method and the more complete method. These show that the simplified method is sufficiently reliable for designed calculations.

FORMULATION OF THE PROBLEM

Assume a cylindrical pipe of length L_t and internal constant diameter, D , to which heat is supplied uniformly from outside. Suppose that in the (water) one-dimensional stream there are only three separate regions: in the first there is a subcooled boiling, in the second a bulk boiling, and in the third there are superheating conditions. It is well known that a rigorous analysis may allow the identification of other regions: this is a first approximation.

Assuming L_1 , L_2 and L_3 as lengths of the three regions (where obviously $L_1 + L_2 + L_3 = L_t$), and Q_1 , Q_2 and Q_3 as heat quantities per unit time supplied by the surroundings to each of them. Assuming that the heat quantity which is given to each length unit Q_L is constant, the result is:

$$Q_L L_t = GA(h_u - h_i). \quad [2]$$

The assumption that Q_L is constant is not restrictive, as the same calculations can be developed with Q_L as function of the length, i.e. for a nuclear reactor the trend of Q_L would be roughly cosinusoidal.

The assumption of Q_L as a constant is physically true enough for many types of boilers and heat exchangers, where the heat is mostly given by very hot fumes, flames, etc. Considering the pipe horizontal, the expressions for the pressure losses can be now drawn. In the first region the flow is one-phase, so that neglecting the pressure losses due to acceleration, we have only the losses due to friction, i.e.

$$\Delta P_1 = \frac{f_1 L_1}{\rho_1 2D} G^2. \quad [3]$$

In the second region where there is boiling, the increase of the specific volume is no longer negligible.

For an exact evaluation of the losses in the pressure due to friction, it would be suitable to consider the slip model with the Martinelli-Nelson friction multiplier, drawing it from the existing graphs, where ϕ_{MN}^2 is plotted as a function of the quality of the outlet flow and of the pressure.

Nevertheless, to reach the result we preferred to make an approximation that can also be gross, but that enables a treatment facility and a real calculation possibility that is valid in most cases, i.e. we suppose that the pressure losses due to friction are caused by the addition of the losses due to the liquid and of the losses due to the steam calculated separately. Practically, we are using an arbitrary superimposition principle that, up to 20 yr ago, was used for boiler calculations; We obtain:

$$\Delta P_{2f} = \frac{f_1 A \lambda}{\rho_1 6 D Q_L} G^3 + \frac{f_2 A \lambda}{\rho_2 6 D Q_L} G^3 \quad [4]$$

that represents the pressure drops due to friction in the second region.

As far as the pressure losses due to acceleration are concerned (El-Wakil 1971), we have:

$$\Delta P_{2a} = G^2(v'' - v') \quad [5]$$

with v' and v'' as the specific volume of the saturated fluid and of the saturated dry steam.

The losses due to friction in the third region are:

$$\Delta P_{3f} = \frac{f_2 L_t}{\rho_2 2D} G^2 - \frac{f_2 A (h'' - h_i)}{\rho_2 2 D Q_L} G^3 \quad [6]$$

where for ρ_2 we choose the value $1/v''$, although it would be preferable to introduce

$$\rho_2' = \frac{1}{v_2'} = \frac{2}{(v_u + v'')} \quad [7]$$

The pressure drops due to acceleration in the third region are:

$$\Delta P_{3a} = G^2(v_u - v'') \quad [8]$$

where v_u is specific volume of the superheated steam at the outlet, and is not known, since h_u is unknown. To determine this quantity we assume that the law for the perfect gases is valid for the superheated steam: $Pv_u = RT_u$. The assumption that the perfect gas law is valid for the whole third region is a remarkable one.

From this we obtain:

$$v_u = \frac{RQ_L L_t}{PC_{p2}GA} - \frac{R(h'' - h_i)}{PC_{p2}} + \frac{R}{P} T_s, \quad [9]$$

and therefore [8] becomes:

$$\Delta P_{3a} = \frac{RQ_L L_t}{PAC_{p2}} G + \left(\frac{R}{P} T_s - \frac{R(h'' - h_i)}{PC_{p2}} - v'' \right) G^2. \quad [10]$$

To sum up, with some simplifications, the total pressure drop can be expressed with a parabolic equation of the third degree, such as:

$$\Delta P(G) = K_1 G + K_2 G^2 + K_3 G^3, \quad [11]$$

with

$$K_1 = \frac{R Q_L L_t}{P A C_{p2}} = \frac{Q_L L_t}{A P} \frac{K - 1}{K}, \quad [11a]$$

$$K_2 = \frac{f_2}{\rho_2} \frac{L_t}{2D} + \frac{R}{P} \left(T_s - \frac{h'' - h_i}{C_{p2}} \right) - v'', \quad [11b]$$

$$K_3 = \frac{f_1}{\rho_1} \frac{A}{6DQ_L} [3(h' - h_i) + \lambda] + \frac{f_2}{\rho_2} \frac{A}{6DQ_L} [\lambda - 3(h'' - h_i)]. \quad [11c]$$

The static instability can be studied using [11] on the basis of Ledinegg's criterion[1].

In order that the equation [11] is really utilizable, it is necessary to make some assumptions, bearing in mind that the equations have been obtained by making the hypothesis of the existence of three regions in the pipe. It is clear that as the flow rate increases, under the same transmitted heat quantity, the fraction of pipe containing is superheated steam decreases, because the specific energy transmitted to the fluid decreases.

The point where the third region disappears can also be reached. We indicate with G_s the value of the flow rate when that happens. The formula we have found is significant only up to G_s ; after that we have to find another formula for the same pipe, where there are only two regions, one subcooled and one with bulk boiling. For this equation a limit also exists which is reached when G is such that the transmitted heat is barely sufficient to bring the liquid to the state of saturation.

This situation happens for a boundary value of the flow rate that we indicate with G_e (for $G = G_e$ we have $L_2 = 0$).

For $G > G_e$ there will be only subcooled water in the pipe and consequently there will be only one region.

For clarity, we indicate with ΔP_A , ΔP_B , ΔP_C , the equations that express the pressure losses in the case of a pipe divided into three regions, into two regions and into one region respectively.

ΔP_A , case with pipe divided into three regions, corresponds to ΔP of the expression [11].

With similar reasoning to the ones developed for ΔP_A , for $G_s \leq G \leq G_e$ we get ΔP_B and so on for ΔP_C .

As known, using the explained method, within the limits of the introduced hypotheses, we can describe the instability phenomenon joining three different equations relevant to as many value-intervals for the specific flow rate: the curve that we get is the one in figure 1 and it has the well-known shape, also proposed by Ledinegg, to determine the instability criterion.

SOLUTIONS TO AVOID THE INSTABILITY

From Ledinegg's criterion [1] we understand immediately that to prevent aperiodic instability, it is necessary to act such that in the characteristic curve "flow rate vs. drop in pressure" of the two-phase flow, there will be no part with negative slope, i.e. it is necessary to use a "means" (stabilizing effect), the introduction of which from a theoretical point of view, gives rise to the deformation of the curve of figure 1, in order not to have any part with a negative slope. Practically this can be obtained by putting an orifice in the pipe inlets, i.e. mechanical diaphragms with suitable dimensions, shape and materials, such as to give the stabilizing effect and not to be corroded by the fluid.

Theoretically this means to introduce a concentrated pressure loss that modifies the curve of figure 1 in the desired way. This also results in an energy loss and therefore it is important to optimize this orifice.

FORMULATION OF A NEW SIMPLIFIED METHOD FOR THE PROBLEM TREATMENT AND SOLUTION

There are formulae, two of which are well known, i.e. Rankine's for solids compressively loaded, and Le Duc's for ballistics, that describe a physical law knowing only what it imposes in boundary conditions and having more or less correct notions on what it requests in intermediate situations.

If it is sufficient for the law to have only an approximate description, it is easy to find

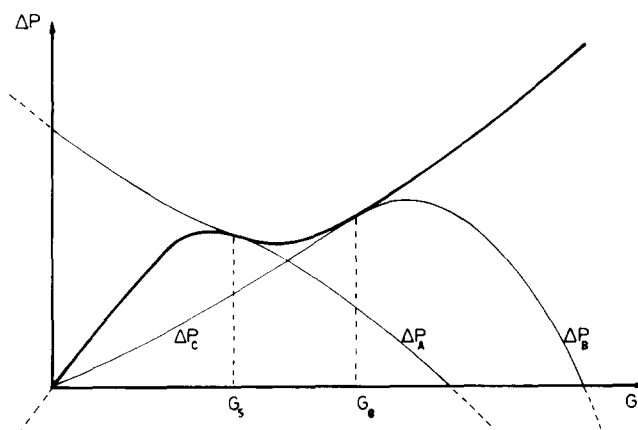


Figure 1.

mathematical expressions that satisfy the boundary conditions and that are possible in the intermediate cases.

Obviously a control of these expressions is also necessary to know what they really represent. Through this criterion we will try to achieve a formulation able to solve in an easy way the discussed problem.

Unfortunately in this case we have great difficulties, because the function we are looking for, representing the instability phenomenon, must have within itself a maximum and a minimum value. These values need to be located in the most exact way, as there is no theory giving the exact coordinates of these points. Consequently we shall have to take into consideration some ordinary points, and this undoubtedly reduces the precision of the proposed method.

It is clear that, the closer the coordinates, chosen at random, are to the region of possible instability, the better the curve, obtained by following the reasoning done by Le Duc, will be able to represent the phenomenon. Luckily, in this case, the used method offers a simple self-control possibility, hence we have a verification of the results. It is known that in a two-phase flow there must be:

$$\lim_{G \rightarrow 0} \Delta P = \lim_{G \rightarrow 0} K'_1 G^2 \tag{12}$$

$$\lim_{G \rightarrow \infty} \Delta P = \lim_{G \rightarrow \infty} K'_2 G^2. \tag{13}$$

The equation we are going to use for a quick solution of the problem must match at the origin and at the infinite, the quadratic curves of [12] and [13]. This is achieved if we assume:

$$\Delta P = K'_1 G^2 \frac{1 + (G/G_f)^m (K'_1/K'_2)^{-1/2}}{1 + (G/G_f)^m (K'_1/K'_2)^{+1/2}} \tag{14}$$

where it is necessary to determine the parameters m and G_f and where we have:

$$K'_1 = \frac{C_c}{2\rho_1} + f_2 \frac{L_1}{D} \frac{v_{2s}}{2} + v_u - v \tag{15}$$

$$K'_2 = \frac{C_c}{2\rho_1} + f_2 \frac{L_1}{D} \frac{v_l}{2}. \tag{16}$$

Note that K'_1 is the valid coefficient for small flow rates ($G \rightarrow 0$), that vaporize as soon as they enter the pipes, and is given by the sum of the concentrated loss of pressure at the inlet (the determination of which is very important because the system can thus be stabilized) plus the pressure loss due to friction on the superheated steam, plus the pressure loss due to the change of momentum; in the same way K'_2 is the valid coefficient for very high flow rates ($G \rightarrow \infty$), that do not vaporize and is given by the sum of the concentrated pressure loss at the inlet, plus the loss due to friction of the liquid along the pipe.

For G_f we see immediately that it is a particular value of G for which the proportionality coefficient K' between ΔP and G^2 is:

$$K' = \sqrt{(K'_1 K'_2)}. \tag{17}$$

It is necessary to give an exact value to G_f and m . The system, the stability of which has to be guaranteed, must be wholly known excluding the inlet loss coefficient C_c that is determined by using the suggested method, in order to guarantee the stability of the system itself.

We calculate then $\Delta P'_A$ and $\Delta P'_B$ for $G = G_A$ and $G = G_B$ respectively, using for instance, the foregoing method or some other less simplified method. Introducing these values in [14] we have:

$$\frac{G_f}{G_A} = \left[\left(\frac{K'_1}{K'_2} \right)^{1/2} \frac{K'_A - K'_2}{K'_1 - K'_A} \right]^{1/m} \quad [18]$$

$$m = \frac{\log(K'_1 - K'_B) + \log(K'_A - K'_2) - \log(K'_B - K'_2) - \log(K'_1 - K'_A)}{\log G_B - \log G_A} \quad [19]$$

with

$$K'_A = \frac{\Delta P'_A}{G_A^2} \quad [20]$$

$$K'_B = \frac{\Delta P'_B}{G_B^2}.$$

From [19] we can deduce that m is the function of all the properties of the fluid and of the circuit characteristics (i.e. of K'_1 , K'_2 , K'_A , K'_B) but not of the concentrated losses at the inlet, as they have a constant additional weight in K'_1 , K'_2 , K'_A , K'_B for which C_c disappears from (19): the very useful practical result is that whatever value we give to C_c we need not recalculate m .

The stability (or better the neutral equilibrium) is hardly ensured when the [14] has neither maximum nor minimum values, excluding obviously the ones for the extreme values of G , but there is a point of flex, and for this it is necessary that

$$\sqrt{\left(\frac{K'_1}{K'_2} \right)} = \frac{m+2}{m-2} \quad \text{with } m > 2. \quad [21]$$

This is the wanted relation, that, with m known from [19], enables solution of the value of K'_1/K'_2 at the "stability boundary". Then we have:

$$\sqrt{\left(\frac{K'_1}{K'_2} \right)} \gtrless \frac{m+2}{m-2} \quad [22]$$

where, when the sign $>$ is valid, there is the possibility of unstable motion, and when the sign $<$ is valid there is stable motion.

The expression [14] represents a curve that at the origin, at the infinite and in two points chosen at random is correct and has a "structure" apt to represent the phenomenon taken in examination: we must verify if that is always true or not. As mentioned before, the method used to determine expression [14] and consequently this diagram flow rate-pressure drop for a two-phase flow offers also a very easy control possibility, calculating the pressure loss for a flow rate different from the two foregoing ones, which have been already used. Assume:

$$\Delta P(G_c) = \Delta P'_c, \quad [23]$$

we have now the possibility of determining three values of m with [19].

If the three values of m are approximately equal to each other, the problem is solved, otherwise we can determine an optimal value with the method of the minimum squares.

Moreover a precision index of the approximate method is given by the possible changes of the values, calculated with the different values of m , of the concentrated pressure loss at the beginning of the pipe, introduced to eliminate the possible instability of the two-phase flow.

Making a ratio between [15] and [16] and drawing C_c we have:

$$C_c = 2\rho_1 \frac{f_2(L_t/D)(n_{2s}/2) + v_{ss} - v_l - (K'_1/K'_2)(f_1(L_t v_l/2D))}{(K'_1/K'_2) - 1} \tag{24}$$

Using suitable tables, we can go back to the ratio between the orifice area and the pipe section.

APPLICATION

Reference is made to a sodium–water heat exchanger used to produce steam in case of fast nuclear reactors (see figure 2). The water flows through the area section, corresponding to the sum of the areas of the three circular sectors, located by the intersection of the triangle (representing the elementary cell) with the three circumferences that represent in plan the sections of pipes; the corresponding sodium flows through the area section equal to that of the triangle, less the above mentioned circular sectors.

For the calculations we use the following values: $D = 10$ mm; $D_{ex} = 16$ mm; $l = 16$ mm; $S_A =$ area of the section relevant to the sodium flow = 343 mm²; operating pressure, water side = 180 atm; sodium inlet and outlet temperature $T_{MN} = 550^\circ\text{C}$ and $T_{mN} = 350^\circ\text{C}$, respectively; the two pinches with the same value $\Delta T_0 = 15^\circ\text{C}$.

From a thermal balance, referred to the water boiling and superheating pipe part and to the respective sodium pipe part we have:

$$\frac{W_A}{W_N} = 0.195.$$

The value of this ratio expresses the water mass flow rate that is necessary to let a Kg of sodium make the required temperature change. From the thermal balance relevant to the economizer part we have: $T_1 \approx 289^\circ\text{C}$. From the thermal balance for the superheating part we have $T'' = 492.5^\circ\text{C}$. Drawing now the different heat transfer coefficients using the formulae given by Friedland (1971) we have:

$$h_e = 31.6 \text{ kJ s}^{-1} \text{ m}^{-2}\text{C}^{-1} \text{ (sodium side)}$$

and the corresponding thermal resistance is

$$R_e = 1/h_e = 0.0316 \text{ s m}^2\text{C/kJ};$$

the wall thermal resistance is:

$$R_p = 0.177 \text{ kJ}^{-1} \text{ m}^2 \text{ s}^\circ\text{C};$$

$h_s =$ transfer heat coefficient due to the dirt:

$$h_s = 12.54 \text{ kJ m}^{-2} \text{ s}^{-1}\text{C}^{-1}$$

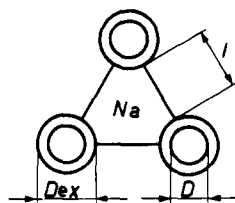


Figure 2.

and the thermal resistance:

$$R_s = 0.08 \text{ kJ}^{-1} \text{ m}^2 \text{ s}^\circ\text{C};$$

the heat transfer coefficient at the water side are three, one for each flow region of the pipe:

$$h_{1s} = 12.93 \text{ kJ s}^{-1} \text{ m}^{-2}\text{C}^{-1},$$

$$h_{2e} = h_{2\text{conv}} + h_{2\text{bo}} = 10.95 + 5.43 = 16.38 \text{ kJ s}^{-1} \text{ m}^{-2}\text{C}^{-1},$$

$$h_{3v} = 8.96 \text{ kJ s}^{-1} \text{ m}^{-2}\text{C}^{-1},$$

and respectively we have the thermal resistances:

$$R_{1s} = 0.077,$$

$$R_{2e} = 0.112,$$

$$R_{3v} = 0.061,$$

all in $\text{kJ}^{-1} \text{ s m}^2\text{C}$. The global coefficient of thermal exchange can be calculated for the three regions, and they are:

$$U_{\text{tot1}} = 2.73, \quad U_{\text{tot2}} = 2.86, \quad U_{\text{tot3}} = 2.5 \text{ all in kW/m}^2\text{C}.$$

Now the total length of pipe and the partial lengths of the three regions into which the pipe is divided can be determined; always by means of a thermal balance we have:

$$L_1 = 3.1 \text{ m}, \quad L_2 = 9.69 \text{ m}, \quad L_3 = 5.29 \text{ m}, \text{ from which } L_t = 18.08 \text{ m}.$$

These values have been found supposing a flow rate $W_A = 0.0613 \text{ kg/s}$. In the proposed method one must choose at least another flow rate and a further one for a control, i.e. W_B and W_C . Assume $W_B = \sqrt{3}$. $W_A = 0.0106 \text{ kg/s}$.

According to the new situation, one has to calculate the following unknown quantities: T' , T'' , T_u , T_{mN} , L_1 , L_2 , L_3 , indicate with L_{1B} , L_{2B} , L_{3B} , particularly we have:

$$L_{1B} = 3.304 \text{ m}, \quad L_{2B} = 11.634 \text{ m}, \quad L_{3B} = 3.142 \text{ m},$$

hence $L_{tB} = 18.08$. Assuming

$$W_c = (1/\sqrt{3})W_A = 0.0354 \text{ kg/s},$$

consequently we have:

$$L_{1c} = 2.176 \text{ m}; \quad L_{2c} = 6.487 \text{ m}; \quad L_{3c} = 9.5 \text{ m} \text{ and } L_{tc} = 18.16 \text{ m}.$$

Calculating now the pressure losses $\Delta P'_A$, $\Delta P'_B$, $\Delta P'_C$ (caused by the three flow rates W_A , W_B , W_C) we obtain:

$$K'_A = \frac{\Delta P'_A}{G_A^2} = 1.32 \cdot 10^{-2} \text{ s}^2 \text{ m}^2 \text{ kg}^{-1},$$

$$K'_B = \frac{\Delta P'_B}{G_B^2} = 0.749 \cdot 10^{-2} \text{ s}^2 \text{ m}^2 \text{ kg}^{-1},$$

$$K'_C = \frac{\Delta P'_C}{G_C^2} = 1.87 \cdot 10^{-2} \text{ s}^2 \text{ m}^2 \text{ kg}^{-1},$$

$$K'_1 = 2.40 \cdot 10^{-2} \text{ s}^2 \text{ m}^2 \text{ kg}^{-1} \dagger$$

$$K'_2 = 2.16 \cdot 10^{-3} \text{ s}^2 \text{ m}^2 \text{ kg}^{-1} \dagger.$$

Making the calculations, from [19] we have:

$$m_{AB} = 2.075; m_{BC} = 2.06; m_{AC} = 2.05.$$

From these results one can consider that a sufficiently correct value of m is 2.06, hence in [22] we have immediately:

$$\sqrt{\left(\frac{K'_1}{K'_2}\right)} \ll \frac{m+2}{m-2}.$$

According to the criterion which has been stated with the proposed method, one can assert that the motion is always stable, i.e. in the curve representing the phenomenon there is no part with negative slope (and no point of flex either). In figure 3 there is the diagram of $\Delta P = \Delta P(G)$, which confirms the above.

Note that the two parabolas thinly drawn represent the trend of (12) and (13), that have the aforesaid significance (a verification of that can be drawn from figure 3).

In figure 3 the curve $\Delta P = \Delta P(G)$ marked with a dashed line, is calculated for the same values taken in consideration in this example, but using the first method of the note, with all its simplifications.

It is visible that the concordance is excellent: the values of ΔP , with G lower than about 2000, are slightly higher than the other ones (a few per cent, maximum about 6 per cent only for a limited range of values around $G \approx 800$).

As this difference is really minimal we can state that the second proposed method is certainly acceptable.

A further comparison has been made with the experimental data given by Ferrari (1978) thus obtaining another confirmation that with the chosen values for the considered quantities, there is no instability in the two-phase flow.

CONCLUSIONS

In the first method proposed, similar to one used in the past, a series of simplifications are used that enables the phenomenon of static instability to be treated in an elementary way, thus obtaining a third degree parabola, an equation that is only partially adequate (e.g. for $G \rightarrow 0$ and $G \rightarrow \infty$ it does not satisfy the desired requirements). Moreover it is necessary to make three equations, depending on the value of G , and to join them.

The second method proposed is a manual one, but, at least in the most common and easiest cases, is able to easily give the required results, including the complete trend of the phenomenon. Moreover there is the possibility of controlling the reliability of the obtained data. We conclude by pointing out the importance of knowing whether and when this type of aperiodic instability of the two-phase flow happens (which instability reveals itself through a flow rate change) in order to eliminate it and to avoid all the consequences explained in the preface to this note.

†Note that in K'_1 and K'_2 the term of concentrated pressure losses caused by a possible stabilizing diaphragm, at the beginning of the pipe, is not included.

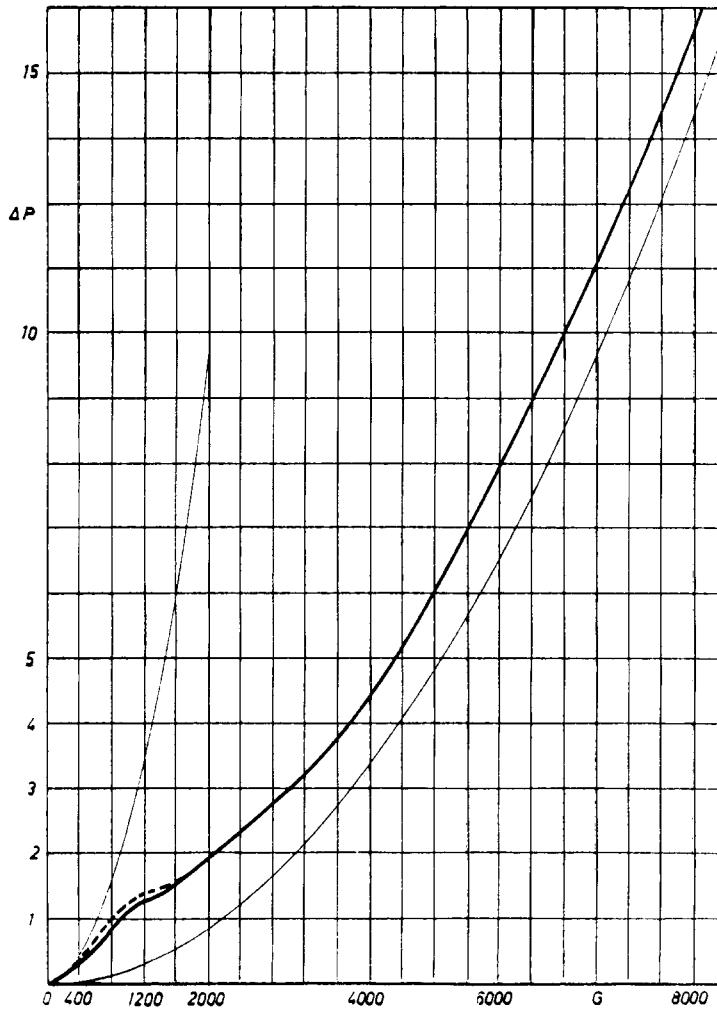


Figure 3.

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